Estadística

1) $\overline{X} \to N(\mu,50)$, el nivel de confianza es del 95% , es decir, $z_{\alpha/2} = 1.96$. El error ha de ser menor que 5 horas

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow 5 = 1.96 \cdot \frac{50}{\sqrt{n}} = \frac{98}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{98}{5} = 19.6 \Rightarrow n = (19.6)^2 = 384.16$. Luego $\boxed{n = 385}$

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2) Calculamos la media muestral $\overline{X} = \frac{100+150+90+70+105+200+120+80+75}{9} = 110$

Sabemos que $X \to N(\mu,12)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Como $n = 9$ y $\sigma = 12$

El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(110 - 1.96 \cdot \frac{12}{\sqrt{9}}, 110 + 1.96 \cdot \frac{12}{\sqrt{9}}\right) =$

$= (110 - 1.96 \cdot 4, 110 + 1.96 \cdot 4) = (110 - 7.84,\ 110 + 7.84) = \boxed{(102.16, 117.84)}$

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3) a) Como $X \to N(\mu,\sigma)$ y las muestras son de tamaño n, $\boxed{\overline{X} \to N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)}$

b) Si $n = 4$ y $X \to N(165,12) \Rightarrow \overline{X} \to N\left(165, \frac{12}{\sqrt{4}}\right) = N(165,6)$. Así: $p(\overline{X} > 173.7) = p\left(\frac{\overline{X} - \mu}{\sigma} > \frac{173.7 - 165}{6}\right) =$

$= p\left(z > \frac{173.7 - 165}{6}\right) = p\left(z > \frac{8.7}{6}\right) = p(z > 1.45) = 1 - p(z \le 1.45) =$ ver tablas $= 1 - 0.9625 = \boxed{0.0735}$

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4) a) Como $X \to N(\mu,3)$ y las muestras son de tamaño 16, $\overline{X} \to N\left(\mu, \frac{3}{\sqrt{16}}\right) = \boxed{N(\mu, 0.75)}$

b) $p = 1 - \alpha = 0.99 \Rightarrow z_{\alpha/2} = 2.58$. Así: $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow 1 = 2.58 \cdot \frac{3}{\sqrt{n}} = \frac{7.74}{\sqrt{n}} \Rightarrow \sqrt{n} = 7.74 \Rightarrow n = (7.74)^2 = 59.90$

Luego $\boxed{n = 60}$

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5) $X \to N(\mu,50000)$, $\overline{X} = 225000$, $\overline{X} \to N\left(\mu, \frac{50000}{\sqrt{16}}\right) = N(\mu,12500)$

El nivel de confianza es del 99%, luego $z_{\alpha/2} = 2.58$. Como $n = 16$. El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$

Sustituyendo: $= \left(225000 - 2.58 \cdot \frac{12}{\sqrt{16}}, 225000 + 2.58 \cdot \frac{12}{\sqrt{16}}\right) = (225000 - 2.58 \cdot 3, 225000 + 2.58 \cdot 3) = \boxed{(192750, 257250)}$

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6) $X \to N(\mu, 0.6)$, $\overline{X} = 7.4$, $n = 30$, $\overline{X} \to N\left(\mu, \dfrac{0.6}{\sqrt{30}}\right) = N(\mu, 0.109)$

a) El nivel de confianza es del 99%, luego $z_{\alpha/2} = 2.58$. Como $n = 16$. El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right)$

Sustituyendo: $= \left(7.4 - 2.58 \cdot \dfrac{0.6}{\sqrt{30}}, 7.4 + 2.58 \cdot \dfrac{0.6}{\sqrt{30}}\right) = (7.54 - 2.58 \cdot 0.109, 7.54 + 2.58 \cdot 0.109) = \boxed{(7.25, 7.68)}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Así como $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}} \Rightarrow 1.96 \cdot \dfrac{0.6}{\sqrt{n}} \le 0.3 \Rightarrow 0.3\sqrt{n} \ge 1.176 \Rightarrow$

$\Rightarrow \sqrt{n} \ge \dfrac{1.176}{0.3} = 3.92 \Rightarrow n \ge (3.92)^2 = 15.36 \Rightarrow \boxed{n = 16}$

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7) Calculamos primero la media muestral: $\overline{X} = \dfrac{7.91 + 7.94 + 7.90 + 7.93 + 7.86 + 7.91}{6} = 7.913$

a) $X \to N(\mu, 0.02)$, $\overline{X} = 7.913$, $n = 6$, $\overline{X} \to N\left(\mu, \dfrac{0.02}{\sqrt{6}}\right) = N(\mu, 0.08)$

El nivel de confianza es del 98%, luego $1 - \alpha = 0.98 \Rightarrow \alpha = 0.02 \Rightarrow \dfrac{\alpha}{2} = 0.01$. Así: $p(z \le z_{\alpha/2}) = 1 - \dfrac{\alpha}{2} = 0.99$. Mirando en tablas, $z_{\alpha/2} = 2.33$. Intervalo $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(7.913 - 2.33 \cdot \dfrac{0.02}{\sqrt{6}}, 7.913 + 2.33 \cdot \dfrac{0.02}{\sqrt{6}}\right) =$

$= \boxed{(7.89, 7.92)}$.

b) El nivel de confianza es del 98%, luego $z_{\alpha/2} = 2.33$. Así como $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}} \Rightarrow 2.33 \cdot \dfrac{0.02}{\sqrt{n}} \le 0.01 \Rightarrow$

$\Rightarrow 2.33 \cdot \dfrac{0.02}{0.01} \le \sqrt{n} \Rightarrow \sqrt{n} \ge 4.66 \Rightarrow n \ge (4.66)^2 = 21.71 \Rightarrow \boxed{n = 22}$

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8) $X \to N(\mu, 10)$, $\overline{X} = 35$, $n = 50$, $\overline{X} \to N\left(\mu, \dfrac{10}{\sqrt{50}}\right) = N(\mu, 0.70)$

El nivel de confianza es del 99%, luego $z_{\alpha/2} = 2.58$. El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right)$

$= \left(35 - 2.58 \cdot \dfrac{10}{\sqrt{50}}, 35 + 2.58 \cdot \dfrac{10}{\sqrt{50}}\right) = \boxed{(31.35, 38.64)}$

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9) $X \to N(50, 6)$,

a) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Como $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}} \Rightarrow 1.96 \cdot \dfrac{6}{\sqrt{n}} \le 2 \Rightarrow 2\sqrt{n} \ge 11.76 \Rightarrow$

$\Rightarrow \sqrt{n} \geq 5.88 \Rightarrow n \geq 34.57 \Rightarrow \boxed{n = 35}$

b) El nivel de confianza es ahora del 90%, luego $z_{\alpha/2} = 1.65$. Análogamente que en a) $1.65 \cdot \frac{6}{\sqrt{n}} \leq 2 \Rightarrow 2\sqrt{n} \geq 9.9 \Rightarrow$

$\Rightarrow \sqrt{n} \geq 4.95 \Rightarrow n \geq 24.50 \Rightarrow \boxed{n = 25}$

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10) El nivel de confianza es del 99%, luego $z_{\alpha/2} = 2.58$. Como $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 0.01 \Rightarrow 2.58 \cdot \frac{0.05}{\sqrt{n}} \leq 0.01 \Rightarrow$

$\Rightarrow \sqrt{n} \geq \frac{2.58 \cdot 0.05}{0.01} = 12.84 \Rightarrow n \geq 166.4 \Rightarrow \boxed{n = 167}$

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11) $X \to N(\mu, 2)$, $\overline{X} = 6.5$, $n = 10$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(6.5 - 1.96 \cdot \frac{2}{\sqrt{10}}, 6.5 + 1.96 \cdot \frac{2}{\sqrt{10}}\right) = \boxed{(5.26, 7.73)}$

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12) $X \to N(\mu, 15)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 3 \Rightarrow 1.96 \cdot \frac{15}{\sqrt{n}} \leq 3 \Rightarrow \sqrt{n} \geq \frac{1.96 \cdot 15}{3} = 9.8 \Rightarrow n \geq 96.04 \Rightarrow \boxed{n = 97}$

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13) a) $X \to N(\mu, 1500)$, $\overline{X} = 20000$, $n = 150$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(20000 - 1.96 \cdot \frac{1500}{\sqrt{150}}, 20000 + 1.96 \cdot \frac{1500}{\sqrt{150}}\right) = \boxed{(19759.81, 20240.19)}$

b) El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 142 \Rightarrow 1.645 \cdot \frac{1500}{\sqrt{n}} \leq 142 \Rightarrow \sqrt{n} \geq \frac{1.645 \cdot 1500}{142} = 17.37 \Rightarrow n \geq 301.95 \Rightarrow \boxed{n = 302}$

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14) a) $X \to N(10, 2)$, $n = 25$, $\overline{X} \to N\left(10, \frac{2}{\sqrt{25}}\right) = N(10, 0.4)$

$p(\overline{X} \leq 9) = p\left(\frac{\overline{X} - \mu}{\sigma} \leq \frac{9 - 10}{0.4}\right) = p(z \leq -2.5) = 1 - p(z \leq 2.5) = 1 - 0.9938 = \boxed{0.0062}$

b) Como $X \to N(10, 2)$, si $n = 64$, $\overline{X} \to N\left(10, \frac{2}{\sqrt{64}}\right) = N\left(10, \frac{2}{8}\right) = \boxed{N(10, 0.25)}$

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15) a) Calculamos la media muestral $\overline{X} = \frac{255 + 85 + 120 + 290 + 80 + 80 + 275 + 290 + 135}{9} = 178.8$

Sabemos que $X \to N(\mu,100)$. El nivel de confianza es del 98%, luego $z_{\alpha/2}=2.33$. Como $n=9$ y $\sigma=100$

El intervalo de confianza será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=\left(178.8-2.33\cdot\frac{100}{\sqrt{9}},178.8+2.33\cdot\frac{100}{\sqrt{9}}\right)=$

$=(178.8-2.33\cdot 33.3,178.8+2.33\cdot 33.3)=(178.8-77.58,178.8+77.58)=\boxed{(101.2,256.4)}$

b) El nivel de confianza es del 99%, luego $z_{\alpha/2}=2.58$

$$E=z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\le 50\Rightarrow 2.58\cdot\frac{100}{\sqrt{n}}\le 50\Rightarrow\sqrt{n}\ge\frac{258}{50}=5.16\Rightarrow n\ge 26.62\Rightarrow\boxed{n=27}$$

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16) Calculamos la media muestral $\overline{X}=\frac{88+90+90+86+87+88+91+92+89}{9}=89$

Sabemos que $X\to N(\mu,1.8)$. El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$. Como $n=9$ y $\sigma=1.8$

El intervalo de confianza será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=\left(89-1.96\cdot\frac{1.8}{\sqrt{9}},89+1.96\cdot\frac{1.8}{\sqrt{9}}\right)=$

$=(89-1.96\cdot 0.6,89+1.96\cdot 0.6)=(89-1.17,89+1.17)=\boxed{(87.83,90.17)}$

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17) $X\to N(\mu,\sqrt{60})$. El nivel de confianza es del 90%, luego $z_{\alpha/2}=1.645$.

$$E=z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\le 3\Rightarrow 1.645\cdot\frac{\sqrt{60}}{\sqrt{n}}\le 3\Rightarrow\sqrt{n}\ge\frac{1.645\cdot 7.74}{3}=4.24\Rightarrow n\ge 18.03\Rightarrow\boxed{n=19}$$

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18) a) $X\to N(\mu,2)$, $\overline{X}=5$, $n=10000$. El nivel de confianza es del 80%, Calculamos $z_{\alpha/2}$.

$1-\alpha=0.8\Rightarrow\alpha=0.2\Rightarrow\frac{\alpha}{2}=0.1$. Así: $p(z\le z_{\alpha/2})=1-\frac{\alpha}{2}=0.9$. Mirando en tablas, $z_{\alpha/2}=1.28$.

El intervalo de confianza será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=\left(5-1.28\cdot\frac{2}{\sqrt{10000}},5+1.28\cdot\frac{2}{\sqrt{10000}}\right)=$

$=\left(5-1.28\cdot\frac{2}{100},5+1.28\cdot\frac{2}{100}\right)=\boxed{(4.97,5.02)}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$

$$E=z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\le 0.25\Rightarrow 1.96\cdot\frac{2}{\sqrt{n}}\le 0.25\Rightarrow\sqrt{n}\ge\frac{3.92}{0.25}=15.68\Rightarrow n\ge 245.86\Rightarrow\boxed{n=246}$$

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19) $X\to N(\mu,25)$ $p\ge 0.95$, luego $z_{\alpha/2}=1.96$

$$E=z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\le 5\Rightarrow 1.96\cdot\frac{25}{\sqrt{n}}\le 5\Rightarrow\sqrt{n}\ge\frac{1.96\cdot 25}{5}=9.8\Rightarrow n\ge 96.4\Rightarrow\boxed{n=97}$$

20) $X \to N(34.5, 6.9)$, $n = 36$, $\bar{X} \to N\left(34.5, \frac{6.9}{\sqrt{36}}\right) = N\left(34.5, \frac{6.9}{6}\right) = N(34.5, 1.15)$

a) $p(32 \le \bar{X} \le 33.5) = p\left(\frac{32-34.5}{1.15} \le \frac{\bar{X}-34.5}{1.15} \le \frac{33.5-34.5}{1.15}\right) = p(-2.17 < z < -0.86) = p(0.86 < z < 2.17) =$

$= p(z < 2.17) - p(z < 0.86) = 0.985 - 0.805 = \boxed{0.1799}$

b) $p(\bar{X} > 38) = p\left(\frac{\bar{X}-34.5}{1.15} > \frac{38-34.5}{1.15}\right) = p(z > 3.04) = 1 - p(z < 3.04) = \boxed{0}$

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21) $X \to N(\mu, 1)$, $n = 100$, $E = 0.2$, luego $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 0.2 \Rightarrow z_{\alpha/2} \cdot \frac{1}{\sqrt{100}} = 0.2 \Rightarrow z_{\alpha/2} \cdot \frac{1}{10} = 0.2 \Rightarrow z_{\alpha/2} = 2$

$p(z \le z_{\alpha/2}) = 1 - \frac{\alpha}{2} \Rightarrow p(z \le 2) = 1 - \frac{\alpha}{2} \Rightarrow 0.9772 = 1 - \frac{\alpha}{2} \Rightarrow \frac{\alpha}{2} = 0.0228 \Rightarrow \alpha = 0.0456 \Rightarrow 1 - \alpha = 0.9544$

Nivel de confianza: $\boxed{95.44\%}$

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22) $\bar{X} \to N\left(\mu, \frac{\sigma}{\sqrt{100}}\right) = N\left(\mu, \frac{\sigma}{10}\right)$, tenemos que calcular μ y σ

- $p(\bar{X} \le 75) = 0.58 \Rightarrow p\left(\frac{\bar{X}-\mu}{\sigma/10} \le \frac{75-\mu}{\sigma/10}\right) = 0.58 \Rightarrow p\left(z \le \frac{75-\mu}{\sigma/10}\right) = 0.58 \Rightarrow$ mirando tablas: $\frac{75-\mu}{\sigma/10} = 0.21$
- $p(\bar{X} > 80) = 0.04 \Rightarrow p(\bar{X} \le 80) = 1 - 0.04 = 0.96 \Rightarrow p\left(\frac{\bar{X}-\mu}{\sigma/10} \le \frac{80-\mu}{\sigma/10}\right) = 0.96 \Rightarrow p\left(z \le \frac{80-\mu}{\sigma/10}\right) = 0.96 \Rightarrow$

mirando tablas: $\frac{80-\mu}{\sigma/10} = 1.76$

Tenemos que resolver el sistema de ecuaciones: $\left\{\begin{array}{l} \frac{75-\mu}{\sigma/10} = 0.21 \\ \frac{80-\mu}{\sigma/10} = 1.76 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{75-\mu}{0.21} = \frac{\sigma}{10} \\ \frac{80-\mu}{1.76} = \frac{\sigma}{10} \end{array}\right\} \Rightarrow$ Igualando:

$\frac{75-\mu}{0.21} = \frac{80-\mu}{1.76} \Rightarrow 1.76 \cdot (75-\mu) = 0.21 \cdot (80-\mu) \Rightarrow 115.2 = 1.55\mu \Rightarrow \boxed{\mu = 74.32}$. Sustituyendo: $\frac{75-74.32}{\sigma/10} = 0.21 \Rightarrow$

$\Rightarrow \sigma = \frac{10 \cdot 0.68}{0.21} \Rightarrow \boxed{\sigma = 32.3}$

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23) $X \to N(\mu, 3)$, $\bar{X} = 5$, $n = 10$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

El intervalo será: $\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(5 - 1.96 \cdot \frac{3}{\sqrt{10}}, 5 + 1.96 \cdot \frac{3}{\sqrt{10}}\right) = \boxed{(3.15, 6.85)}$

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24) Calculamos la media muestral $\overline{X} = \dfrac{33+34+26+37+30+39+26+31+36+19}{10} = 31.1$

Sabemos que $X \to N(\mu,5)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Como $n = 10$ y $\sigma = 5$

El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(31.5 - 1.96 \cdot \dfrac{5}{\sqrt{10}}, 31.5 + 1.96 \cdot \dfrac{5}{\sqrt{10}}\right) =$

$= \boxed{(28, 34.2)}$

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25) $X \to N(60,8)$, $n = 100$

a) $\boxed{\mu = 60}$, $\sigma = \dfrac{8}{\sqrt{100}} = \dfrac{8}{10} \Rightarrow \boxed{\sigma = 0.8}$

b) $p(59 \le \overline{X} \le 61) = p\left(\dfrac{59-60}{0.8} \le \dfrac{\overline{X}-60}{0.8} \le \dfrac{61-60}{0.8}\right) = p(-1.25 < z < 1.25) = 2p(z < 1.25) - 1 = 0.6826$

Así: $100 \cdot 0.68.26 = 68.26$, luego $\boxed{68\, muestras}$

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26) $X \to N(35,5)$, $n = 100$

a) $\boxed{\mu = 35}$, $\sigma = \dfrac{5}{\sqrt{100}} = \dfrac{5}{10} = 0.5 \Rightarrow \boxed{\sigma^2 = 0.25}$

b) $p(36 \le \overline{X} \le 37) = p\left(\dfrac{36-35}{0.5} \le \dfrac{\overline{X}-35}{0.5} \le \dfrac{37-35}{0.5}\right) = p(2 < z < 4) = p(z < 4) - p(z < 2) = 1 - 0.9772 = \boxed{0.0228}$

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27) Calculamos la media muestral $\overline{X} = \dfrac{33+34+26+37+30+39+26+31+36+19}{10} = 31.1$

Sabemos que $X \to N(\mu,5)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Como $n = 10$ y $\sigma = 5$

El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(31.5 - 1.96 \cdot \dfrac{5}{\sqrt{10}}, 31.5 + 1.96 \cdot \dfrac{5}{\sqrt{10}}\right) =$

$= \boxed{(28, 34.2)}$

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28) $X \to N(\mu,328)$, $\overline{X} = 1248$, $n = 10$. El nivel de confianza es del 98%, luego $z_{\alpha/2} = 2.575$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(1248 - 2.575 \cdot \dfrac{328}{\sqrt{100}}, 1248 + 2.575 \cdot \dfrac{328}{\sqrt{100}}\right) = \boxed{(1163.34, 1332.46)}$

b) $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}} \le 127 \Rightarrow 1.96 \cdot \dfrac{328}{\sqrt{n}} \le 127 \Rightarrow \sqrt{n} \ge \dfrac{1.96 \cdot 328}{127} = 5.06 \Rightarrow n \ge 25.6 \Rightarrow \boxed{n = 26}$

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29) $X \to N(\mu,32)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 10 \Rightarrow 1.96 \cdot \frac{32}{\sqrt{n}} \leq 10 \Rightarrow \sqrt{n} \geq \frac{1.96 \cdot 32}{10} = 6.27 \Rightarrow n \geq 39.93 \Rightarrow \boxed{n = 40}$$

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30) Calculamos la media muestral $\overline{X} = \dfrac{91+68+39+82+55+70+72+62+54+67}{10} = 66$

a) Sabemos que $X \rightarrow N(\mu, 15)$. El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.64$. Como $n = 10$ y $\sigma = 15$

El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(66 - 1.64 \cdot \dfrac{15}{\sqrt{10}}, 66 + 1.64 \cdot \dfrac{15}{\sqrt{10}}\right) =$

$= \boxed{(58.23, 73.77)}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 5 \Rightarrow 1.96 \cdot \frac{15}{\sqrt{n}} \leq 5 \Rightarrow \sqrt{n} \geq \frac{1.96 \cdot 15}{5} = 5.88 \Rightarrow n \geq 34.5 \Rightarrow \boxed{n = 35}$$

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31) $\sigma = 1$, $n = 64$

a) El nivel de confianza es del 98%, luego $z_{\alpha/2} = 2.33$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2.33 \cdot \frac{1}{\sqrt{64}} = 2.33 \cdot \frac{1}{8} = 0.29 < 0.5 \ \boxed{Si}$$

b) El nivel de confianza ahora es del 95%, luego $z_{\alpha/2} = 1.96$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < 0.5 \Rightarrow 1.96 \cdot \frac{1}{\sqrt{n}} < 0.5 \Rightarrow \sqrt{n} > \frac{1.96}{0.5} = 3.92 \Rightarrow n > 15.3 \Rightarrow \boxed{n = 16}$$

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32) $X \rightarrow N(\mu, 1.5)$, $\overline{X} = 5.95$, $n = 10$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(5.95 - 1.96 \cdot \dfrac{1.5}{\sqrt{10}}, 5.95 + 1.96 \cdot \dfrac{1.5}{\sqrt{10}}\right) = \boxed{(5.02, 6.87)}$

b) $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}} \leq 0.5 \Rightarrow 1.96 \cdot \dfrac{1.5}{\sqrt{n}} \leq 0.5 \Rightarrow \sqrt{n} \geq \dfrac{1.96 \cdot 1.5}{0.5} = 5.8 \Rightarrow n \geq 34.5 \Rightarrow \boxed{n = 35}$

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33) Calculamos la media muestral $\overline{X} = \dfrac{46+38+59+29+34+32+38+21+44+34}{10} = 37.5$

a) Sabemos que $X \rightarrow N(\mu, 10)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Como $n = 10$ y $\sigma = 10$

El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(37.5 - 1.96 \cdot \dfrac{10}{\sqrt{10}}, 37.5 + 1.96 \cdot \dfrac{10}{\sqrt{10}}\right) = \boxed{(31.31, 43.69)}$

b) El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 5 \Rightarrow 1.645 \cdot \frac{10}{\sqrt{n}} \le 5 \Rightarrow \sqrt{n} \ge \frac{1.645 \cdot 10}{5} = 3.29 \Rightarrow n \ge 10.81 \Rightarrow \boxed{n = 11}$$

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34) $X \to N(\mu, 55)$, $\overline{X} = 320$, $n = 81$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{55}{\sqrt{81}} \le \frac{1.96 \cdot 55}{9} = 11.97 > 10$, luego no podemos asegurarlo

b) $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < 10 = 1.96 \cdot \frac{55}{\sqrt{n}} \Rightarrow \sqrt{n} \ge \frac{1.96 \cdot 55}{10} = 10.78 \Rightarrow n \ge 116.2 \Rightarrow \boxed{n = 117}$

---

35) Calculamos la media muestral $\overline{X} = \frac{9.1 + 4.9 + 7.3 + 2.8 + 5.5 + 6 + 3.7 + 8.6 + 4.5 + 7.6}{10} = 6$

a) Sabemos que $X \to N(\mu, 2)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Como $n = 10$ y $\sigma = 2$

El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(6 - 1.96 \cdot \frac{2}{\sqrt{10}}, 6 + 1.96 \cdot \frac{2}{\sqrt{10}}\right) =$

$= \boxed{(4.77, 7.23)}$

b) El nivel de confianza es del 98%, luego $z_{\alpha/2} = 2.33$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 1 \Rightarrow 2.33 \cdot \frac{2}{\sqrt{n}} \le 1 \Rightarrow \sqrt{n} \ge \frac{2.33 \cdot 2}{1} = 4.66 \Rightarrow n \ge 21.7 \Rightarrow \boxed{n = 22}$$

---

36) **a)** $X \to N(\mu, 1.32)$, El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 0.5 \Rightarrow 1.96 \cdot \frac{1.32}{\sqrt{n}} \le 0.5 \Rightarrow \sqrt{n} \ge \frac{1.96 \cdot 1.32}{0.5} = 5.16 \Rightarrow n \ge 26.6 \Rightarrow \boxed{n = 27}$$

b) $\mu = 4.36$, $n = 16$. $p(4 \le \overline{X} \le 5) = p\left(\frac{4 - 4.36}{1.32/\sqrt{16}} \le \frac{\overline{X} - 4.36}{1.32/\sqrt{16}} \le \frac{5 - 4.36}{1.32/\sqrt{16}}\right) = p(-1.09 < z < 1.94) = \boxed{0.82}$

---

37) $X \to N(\mu, 9)$, $\overline{X} = 8$, $n = 20$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(8 - 1.96 \cdot \frac{9}{\sqrt{20}}, 8 + 1.96 \cdot \frac{9}{\sqrt{20}}\right) = \boxed{(4.06, 11.9)}$

b) $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \frac{4}{2} \Rightarrow 1.96 \cdot \frac{9}{\sqrt{n}} \le 2 \Rightarrow \sqrt{n} \ge \frac{1.96 \cdot 9}{2} = 8.82 \Rightarrow n \ge 77.9 \Rightarrow \boxed{n = 78}$

---

38) $X \to N(\mu, 0.5)$, $\overline{X} = 10.3$, $n = 9$. El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(10.3 - 1.645 \cdot \frac{0.5}{\sqrt{9}}, 10.3 + 1.645 \cdot \frac{0.5}{\sqrt{9}}\right) = \boxed{(10.03, 10.57)}$

b) Si $p = 1 - \alpha = 0.98 \Rightarrow z_{\alpha/2} = 2.33$

$$E = z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le 0.2 \Rightarrow 2.33\cdot\frac{0.5}{\sqrt{n}} \le 0.2 \Rightarrow \sqrt{n} \ge \frac{2.33\cdot 0.5}{0.2} = 5.82 \Rightarrow n \ge 33.93 \Rightarrow \boxed{n = 34}$$

---

39) $X \to N(\mu,10)$, $\overline{X} = 19$, $n = 256$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right) = \left(19 - 1.96\cdot\frac{10}{\sqrt{256}}, 19 + 1.96\cdot\frac{10}{\sqrt{256}}\right) = \boxed{(17.78, 20.22)}$

b) El nivel de confianza es ahora del 99%, luego $z_{\alpha/2} = 2.58$.

Error máximo estimación: $\frac{|20.22 - 17.78|}{2} = 1.2$

Luego: $E = z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le 1.2 \Rightarrow 2.58\cdot\frac{10}{\sqrt{n}} \le 1.2 \Rightarrow \sqrt{n} \ge \frac{2.58\cdot 10}{1.2} = 21.5 \Rightarrow n \ge 462.25 \Rightarrow \boxed{n = 463}$

---

40) a) $X \to N(\mu,0.5)$, $\overline{X} = 19.84$, $n = 4$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

El intervalo será: $\left(\overline{X} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right) = \left(19.84 - 1.96\cdot\frac{0.5}{\sqrt{4}}, 19.84 + 1.96\cdot\frac{0.5}{\sqrt{4}}\right) = \boxed{(19.35, 20.33)}$

b) Si $p = 1 - \alpha = 0.95 \Rightarrow z_{\alpha/2} = 1.96$

$$E = z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le 0.2 \Rightarrow 1.96\cdot\frac{0.5}{\sqrt{n}} \le 0.2 \Rightarrow \sqrt{n} \ge \frac{1.96\cdot 0.5}{0.2} = 4.9 \Rightarrow n \ge 24.01 \Rightarrow \boxed{n = 25}$$

---

41) $X \to N(\mu,0.5)$, $\overline{X} = 6$, $n = 100$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right) = \left(6 - 1.96\cdot\frac{0.5}{\sqrt{100}}, 6 + 1.96\cdot\frac{0.5}{\sqrt{100}}\right) = \boxed{(5.9, 6.1)}$

b) $E = z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le \frac{1}{2} \Rightarrow 1.96\cdot\frac{0.5}{\sqrt{n}} \le 0.5 \Rightarrow \sqrt{n} \ge \frac{1.96\cdot 0.5}{0.5} = 1.96 \Rightarrow n \ge 3.84 \Rightarrow \boxed{n = 4}$

---

42) a) $X \to N(\mu,320)$, $n = 36$. $p\left(\left|\overline{X} - \mu\right| \ge 50\right) = p\left(-50 \ge \overline{X} - \mu \ge 50\right) = p\left(\mu - 50 \ge \overline{X} \ge \mu + 50\right) =$

$$= p\left(\frac{\mu - 50 - \mu}{320/\sqrt{36}} \ge \frac{\overline{X} - \mu}{320/\sqrt{36}} \ge \frac{\mu + 50 - \mu}{320/\sqrt{36}}\right) = p\left(\frac{-50}{320/6} \ge z \ge \frac{50}{320/6}\right) = p(-0.93 \ge z \ge 0.93) = 2\cdot p(z \ge 0.93) =$$

$$= 2\cdot\left(1 - p(z \le 0.93)\right) = 2\cdot(1 - 0.82) = \boxed{0.34}$$

b) Si $\overline{X} = 4820$, el intervalo será: $= \left(4820 - 1.96\cdot\frac{320}{\sqrt{36}}, 4820 + 1.96\cdot\frac{320}{\sqrt{36}}\right) = \boxed{(4715.6, 4924.4)}$

---

43) $X \to N(\mu,5)$, $n = 100$, el intervalo de confianza es $(173.42, 175.56)$

a) $\overline{X} = \dfrac{173.42 + 175.56}{2} \Rightarrow \boxed{\overline{X} = 174.49}$

b) Error máximo estimación: $\dfrac{|175.56 - 173.42|}{2} = 1.07$, como $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}} \Rightarrow z_{\alpha/2} = \dfrac{E\sqrt{n}}{\sigma} = \dfrac{1.07\sqrt{100}}{5} = 2.14$

Así $z_{\alpha/2} = 2.14 \Rightarrow$ mirando tablas $p(z \le 2.14) = 0.9838 = 1 - \dfrac{\alpha}{2} \Rightarrow \dfrac{\alpha}{2} = 1 - 0.9838 = 0.0162 \Rightarrow \alpha = 0.0324$

Luego, nivel de confianza: $1 - \alpha = 1 - 0.0324 = 0.9676 \Rightarrow \boxed{96.76\%}$

---

44) $X \to N(\mu, 15)$, $\overline{X} = 108$, $n = 9$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(108 - 1.96 \cdot \dfrac{15}{\sqrt{9}}, 108 + 1.96 \cdot \dfrac{15}{\sqrt{9}}\right) = \boxed{(98.2, 117.8)}$

b) Sabemos que $\mu = 110$. Así: $p(\overline{X} > 120) = p\left(\dfrac{\overline{X} - 110}{15/\sqrt{9}} \ge \dfrac{120 - 110}{15/\sqrt{9}}\right) = p\left(z \ge \dfrac{10}{5}\right) = p(z > 2) = 1 - p(z < 2) = \boxed{0.022}$

---

45) a) $X \to N(\mu, 400)$, $n = 100$. Sabemos $p(|\overline{X} - \mu| \le 66) = p(-66 \le \overline{X} - \mu \le 66) = p(\mu - 66 \le \overline{X} \le \mu + 66) =$

$$= p\left(\frac{\mu - 66 - \mu}{400/\sqrt{100}} \le \frac{\overline{X} - \mu}{400/\sqrt{100}} \le \frac{\mu + 66 - \mu}{400/\sqrt{100}}\right) = p\left(\frac{-66}{400/10} \le z \le \frac{66}{400/10}\right) = p(-1.65 \le z \le 1.65) = 2 \cdot p(z \le 1.65) =$$

$= 2 \cdot 0.9505 = 1.901 = 1 - \alpha \Rightarrow \alpha = 1 - 1.901 = 0.901 \Rightarrow \boxed{90.1\%}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 40 \Rightarrow 1.96 \cdot \frac{400}{\sqrt{n}} \le 40 \Rightarrow \sqrt{n} \ge \frac{1.96 \cdot 400}{40} = 19.6 \Rightarrow n \ge 384.16 \Rightarrow \boxed{n = 385}$$

---

46) $X \to N(\mu, 15)$, $\overline{X} = 3 = 180\,\text{min}$, $n = 400$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(180 - 1.96 \cdot \dfrac{15}{\sqrt{400}}, 180 + 1.96 \cdot \dfrac{15}{\sqrt{400}}\right) = \boxed{(178.5, 181.5)}$

b) El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 5 \Rightarrow 1.645 \cdot \frac{15}{\sqrt{n}} \le 5 \Rightarrow \sqrt{n} \ge \frac{1.645 \cdot 15}{5} = 8.225 \Rightarrow n \ge 67.65 \Rightarrow \boxed{n = 68}$$

---

47) Calculamos la media muestral $\overline{X} = \dfrac{1.50 + 1.60 + 1.10 + 0.90 + 1.00 + 1.60 + 1.40 + 0.90 + 1.30 + 1.20}{10} = 1.25$

Sabemos que $X \to N(\mu, 0.09)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

El intervalo de confianza será: $\left(\overline{X} - z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}\right) = \left(1.25 - 1.96 \cdot \dfrac{0.09}{\sqrt{10}}, 1.25 + 1.96 \cdot \dfrac{0.09}{\sqrt{10}}\right) =$

$= \boxed{(1.19, 1.31)}$

b) $p = 0.99 \Rightarrow$ El nivel de confianza es del 99%, luego $z_{\alpha/2} = 2.58$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 0.1 \Rightarrow 2.58 \cdot \frac{0.09}{\sqrt{n}} \le 0.1 \Rightarrow \sqrt{n} \ge \frac{2.58 \cdot 0.09}{0.1} = 2.32 \Rightarrow n \ge 5.39 \Rightarrow \boxed{n = 6}$$

---

48) a) $X \to N(98,15)$, $n = 9$. $p(\overline{X} > 100) = p\left(\frac{\overline{X} - \mu}{15/\sqrt{9}} \le \frac{100-98}{15/\sqrt{9}}\right) = p\left(z > \frac{2}{5}\right) = p(z > 0.4) = 1 - p(z \le 0.4) =$

$= 1 - 0.6554 = \boxed{0.3446}$

b) Nos piden $p(\overline{X} < 104 / \overline{X} > 100) = \frac{p(\overline{X} < 104 \cap \overline{X} > 100)}{p(\overline{X} > 100)} = \frac{p(100 < \overline{X} < 104)}{p(\overline{X} > 100)} =$ tipificando $=$

$$= \frac{p\left(\frac{100-98}{15/3} < \frac{\overline{X}-98}{15/3} < \frac{104-98}{15/3}\right)}{p\left(\frac{\overline{X}-98}{15/3} > \frac{100-98}{15/3}\right)} = \frac{p\left(\frac{2}{5} < z < \frac{6}{5}\right)}{p\left(z > \frac{2}{5}\right)} = \frac{p(0.4 < z < 1.2)}{p(z > 0.4)} = \frac{p(z < 1.2) - p(z < 0.4)}{1 - p(z < 0.4)} = \frac{0.88 - 0.65}{0.35} =$$

$= \frac{0.23}{0.35} = \boxed{0.65}$

---

49) $X \to N(\mu, 10)$, $\overline{X} = 110$, $n = 9$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

a) El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(110 - 1.96 \cdot \frac{10}{\sqrt{9}}, 110 + 1.96 \cdot \frac{10}{\sqrt{9}}\right) = \boxed{(103.46, 116.53)}$

b) $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 5 \Rightarrow 1.96 \cdot \frac{10}{\sqrt{n}} \le 5 \Rightarrow \sqrt{n} \ge \frac{1.96 \cdot 10}{5} = 3.92 \Rightarrow n \ge 15.36 \Rightarrow \boxed{n = 16}$

---

50) a) Calculamos la media muestral $\overline{X} = \frac{26 + 1.27.5 + 31 + 28 + 25.5 + 30.5 + 32 + 31.5}{8} = 29$

Sabemos que $X \to N(\mu, 2.8)$. El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$.

El intervalo será: $\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(29 - 1.645 \cdot \frac{2.8}{\sqrt{8}}, 29 + 1.645 \cdot \frac{2.8}{\sqrt{8}}\right) = \boxed{(27.3, 30.6)}$

b) El nivel de confianza es del 97%, luego $z_{\alpha/2} = 2.17$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 0.9 \Rightarrow 2.17 \cdot \frac{2.8}{\sqrt{n}} \le 0.9 \Rightarrow n \ge 45.56 \Rightarrow \boxed{n = 46}$$

---

51) a) $X \to N(\mu, 45)$. El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

El intervalo de confianza es $(251.6, 271.2) = \left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$

Sumando los extremos del intervalo: $\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} + \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2\bar{X} \Rightarrow \bar{X} = \frac{251.6 + 271.2}{2} \Rightarrow \boxed{\bar{X} = 261.4}$

Como $\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 251.6 \Rightarrow 261.4 - 1.96 \cdot \frac{45}{\sqrt{n}} = 261.4 \Rightarrow \sqrt{n} = \frac{88.2}{9.8} = 9 \Rightarrow \boxed{n = 81}$

b) $X \to N(\mu, 45)$. El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$, $n = 64$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{45}{\sqrt{64}} = \frac{1.645 \cdot 45}{8} = \frac{74.025}{8} \Rightarrow \boxed{E = 9.25}$$

---

52) a) $X \to N(\mu, 3000)$. El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$. $\bar{X} = 48000$, $n = 100$

El intervalo será: $\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(48000 - 1.645 \cdot \frac{3000}{\sqrt{100}}, 48000 + 1.645 \cdot \frac{3000}{\sqrt{100}}\right) =$

$= \boxed{(475065, 484935)}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le 1000 \Rightarrow 1.96 \cdot \frac{3000}{\sqrt{n}} \le 1000 \Rightarrow 1.96 \cdot \frac{3000}{1000} \le \sqrt{n} \Rightarrow \sqrt{n} \ge 5.88 \Rightarrow n \ge 34.47 \Rightarrow \boxed{n = 35}$$

---

53) a) $\bar{X} \to N(\mu, 3)$, $n = 121$. $p(|\bar{X} - \mu| > 0.5) = p(\bar{X} - \mu \ge 0.5) + p(\bar{X} - \mu \le -0.5) = 1 - p(\bar{X} - \mu \le 0.5) + p(\bar{X} - \mu \le -0.5)$

$= 1 - p\left(\frac{\bar{X} - \mu}{3/\sqrt{121}} \le \frac{0.5}{3/11}\right) + p\left(\frac{\bar{X} - \mu}{3/\sqrt{121}} \le \frac{-0.5}{3/11}\right) = 1 - p(z \le 1.83) + p(z \le -1.83) = 1 - p(z \le 1.83) + p(z \ge 1.83) =$

$= 1 - p(z \le 1.83) + 1 - p(z \le 1.83) = 2 - 2p(z \le 1.83) = 2 - 2 \cdot 0.9664 = \boxed{0.0672}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$, $\bar{X} = 7$, $n = 121$

El intervalo será: $\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(7 - 1.96 \cdot \frac{3}{\sqrt{121}}, 7 + 1.96 \cdot \frac{3}{\sqrt{121}}\right) = \boxed{(6.645, 7.535)}$

---

54) $X \to N(3.5, 1.40)$, $n = 49$, luego $\bar{X} \to N\left(3.5, \frac{1.4}{\sqrt{49}}\right) = N\left(3.5, \frac{1.4}{7}\right) = N(3.5, 0.2)$

a) $p(\bar{X} \le 3.37) = p\left(\frac{\bar{X} - 3.5}{0.2} \le \frac{3.37 - 3.5}{0.2}\right) = p(z \le -0.65) = p((z > 0.65) = 1 - p(z \le 0.65) = 1 - 0.7422 = \boxed{0.2578}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$, $\bar{X} = 3.42$, $n = 49$

El intervalo será: $\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) = \left(3.42 - 1.96 \cdot \frac{1.4}{7}, 3.42 + 1.96 \cdot \frac{1.4}{7}\right) = \boxed{(3.028, 3.812)}$

---

55) $X \to N(\mu,1940)$.

a) El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$.

$$E = z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le 100 \Rightarrow 1.96\cdot\frac{1940}{\sqrt{n}} \le 100 \Rightarrow 1.96\cdot\frac{1940}{100} \le \sqrt{n} \Rightarrow \sqrt{n} \ge 38.0 \Rightarrow n \ge 1445.82 \Rightarrow \boxed{n=1446}$$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$, $\overline{X}=12415$, $n=225$

El intervalo será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right) = \left(12415-1.645\cdot\frac{1940}{\sqrt{225}}, 12415+1.645\cdot\frac{1940}{\sqrt{225}}\right)=$

$= \boxed{(122029, 12627.1)}$

---

56) a) $X \to N(\mu,0.4)$. El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$. $\overline{X}=1.75$, $n=400$

El intervalo será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right) = \left(1.75-1.96\cdot\frac{0.4}{\sqrt{400}}, 1.75+1.96\cdot\frac{0.4}{\sqrt{400}}\right)=$

$$=\left(1.75-1.96\cdot\frac{0.4}{20}, 1.75+1.96\cdot\frac{0.4}{20}\right)=(1.75-0.039, 1.75+0.039)=\boxed{(1.71, 1.78)}$$

b) El nivel de confianza es del 90%, luego $z_{\alpha/2}=1.645$.

$$E = z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le 0.02 \Rightarrow 1.645\cdot\frac{0.4}{\sqrt{n}} \le 0.02 \Rightarrow \frac{0.658}{\sqrt{n}} \le 0.02 \Rightarrow \sqrt{n} \ge 32.9 \Rightarrow n \ge 108241 \Rightarrow \boxed{n=1083}$$

---

57) a) $X \to N(\mu,210) \Rightarrow \overline{X} \to N\left(\mu,\frac{210}{\sqrt{64}}\right)=N\left(\mu,\frac{210}{8}\right)$. Por otra parte $p\left(\left|\overline{X}-\mu\right| \ge 22\right)=1-p\left(\left|\overline{X}-\mu\right|<22\right)$ **(*)**

$p\left(\left|\overline{X}-\mu\right|<22\right)=p\left(-22<\overline{X}-\mu<22\right)=p\left(\mu-22<\overline{X}<22+\mu\right)=$tipificando=

$$p\left(\frac{\mu-22-\mu}{210/8}<z<\frac{\mu+22-\mu}{210/8}\right)=p\left(\frac{-22}{210/8}<z<\frac{22}{210/8}\right)=p(-0.84<z<0.84)=2p(z<0.84)-1=$$

$=2\cdot 0.7996-1=0.599$

Sustituyendo en **(*)**: $p\left(\left|\overline{X}-\mu\right| \ge 22\right)=1-p\left(\left|\overline{X}-\mu\right|<22\right)=1-0.599=\boxed{0.40}$

b) $X \to N(\mu,210) \Rightarrow \overline{X} \to N\left(\mu,\frac{210}{\sqrt{64}}\right)=N\left(\mu,\frac{210}{8}\right)$

El nivel de confianza es del 99%, luego $z_{\alpha/2}=2.57$. $\overline{X}=1532$

El intervalo será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=\left(1532-2.57\cdot\frac{210}{8},1532+2.57\cdot\frac{210}{8}\right)=$

$=(1532-67.4,1532+67.4)=\boxed{(1464.6,1599.4)}$

---

58) a) $X\to N(\mu,3)$. El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$. $\overline{X}=36$, $n=48$

El intervalo será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=\left(36-1.96\cdot\frac{3}{\sqrt{48}},36+1.96\cdot\frac{3}{\sqrt{48}}\right)=$

$=\left(36-\frac{5.88}{\sqrt{48}},36+\frac{5.88}{\sqrt{48}}\right)=\boxed{(35.16,36.84)}$

b) El nivel de confianza es del 90%, luego $z_{\alpha/2}=1.645$.

$E=z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\le 1\Rightarrow 1.645\cdot\frac{3}{\sqrt{n}}\le 1\Rightarrow\frac{4.95}{\sqrt{n}}\le 1\Rightarrow\sqrt{n}\ge 4.95\Rightarrow n\ge 24.35\Rightarrow\boxed{n=25}$

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59) a) $X\to N(\mu,3)$. El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$.

El intervalo será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=(16.33,19.27)$ sustituyendo los valores

$\left(\overline{X}-\frac{5.88}{\sqrt{n}},\overline{X}+\frac{5.88}{\sqrt{n}}\right)=(16.33,19.27)$. Igualando los extremos de los intervalos se obtiene el sistema:

$\left\{\begin{array}{l}\overline{X}-\frac{5.88}{\sqrt{n}}=16.33\\ \overline{X}+\frac{5.88}{\sqrt{n}}=19.27\end{array}\right\}$, resolviendo: $\boxed{\overline{X}=17.8}$, $\boxed{n=16}$

b) El nivel de confianza es del 95%, luego $z_{\alpha/2}=1.96$.

$E=z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\le 1\Rightarrow 1.96\cdot\frac{3}{\sqrt{64}}=\frac{5.88}{8}\Rightarrow\boxed{E=0.735}$

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60) a) $X\to N(\mu,16)$. El nivel de confianza es del 98%, luego $z_{\alpha/2}=2.325$. $\overline{X}=169$, $n=625$

El intervalo será: $\left(\overline{X}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=\left(169-2.325\cdot\frac{16}{\sqrt{625}},169+2.325\cdot\frac{16}{\sqrt{625}}\right)=$

$=\left(169-\frac{37.2}{25},169+\frac{37.2}{25}\right)=(169-1.488,169+1.488)=\boxed{(167.512,170.488)}$

b) El nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.645$.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 4 \Rightarrow 1.645 \cdot \frac{16}{\sqrt{n}} \leq 4 \Rightarrow \frac{26.32}{\sqrt{n}} \leq 4 \Rightarrow \sqrt{n} \geq \frac{26.32}{4} = 6.58 \Rightarrow n \geq 43.29 \Rightarrow \boxed{n = 44}$$

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61) Sea $E_1 = 3.290$, el nivel de confianza es del 90%, luego $z_{\alpha/2} = 1.654$. Sea el tamaño muestral n_1

$E_2 = 7.840$, el nivel de confianza es del 95%, luego $z_{\alpha/2} = 1.96$. Sea el tamaño muestral n_2

Sabemos que $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ y que $n_1 = 7500 + n_2$. Así:

- Para el 1º nivel de confianza: $E_1 = 1.645 \cdot \frac{\sigma}{\sqrt{n_1}} \leq 3.290 \Rightarrow \geq \frac{\sigma}{\sqrt{n_1}} \leq \frac{3.290}{1.645} = 2 \Rightarrow \sqrt{n_1} \geq \frac{\sigma}{2} \Rightarrow \boxed{n_1 = \frac{\sigma^2}{4}}$

- Para el 2º nivel de confianza: $E_2 = 1.96 \cdot \frac{\sigma}{\sqrt{n_2}} \leq 7.840 \Rightarrow \frac{\sigma}{\sqrt{n_2}} \leq \frac{7.840}{1.96} = 4 \Rightarrow \sqrt{n_2} \geq \frac{\sigma}{4} \Rightarrow \boxed{n_2 = \frac{\sigma^2}{16}}$

Como $n_1 = 7500 + n_2 \Rightarrow \frac{\sigma^2}{4} = 7500 + \frac{\sigma^2}{16} \Rightarrow \boxed{\sigma = 200}$. Sustituyendo: $\boxed{n_1 = 10000}$ y $\boxed{n_2 = 2500}$
